Title: An Excel-Based System for Managing the Radiation Safety of the $^{131}$I Therapy Patient’s Family, and Others

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Disclaimer: The author has no conflicts of interest to claim.

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Word count of manuscript: 6609

Financial support: none

Short running title: $^{131}$I therapy radiation safety management
Abstract

Excel workbooks are developed that assist the radiation safety counseling of $^{131}$I therapy patients and their families. Use of the workbooks provides individualized guidelines for meeting selected dose limits that avoid overly conservative restrictions to family members and others.

Methods

The mathematical model includes biphasic patient radionuclide retention. The extra-thyroidal component is a cylindrical volume with a diameter corresponding to the patient’s size and includes patient self-absorption, while the thyroidal component is a point source whose transmission is reduced by self-absorption. A separate model in which the thyroid, extra-thyroid, and bladder compartments feed serially from one to the next is developed in order to depict the radionuclide levels within the patient and to estimate the activity entering the environment at each urination.

Results

The system is organized into a set of 4 workbooks: the first to be used with ablation patients prepared using thyrogen, the second with ablation patients prepared by deprivation, the third with hyperthyroid patients, and the fourth is used with the unusual hyperthyroid patient who finds the restrictions to be oppressive and returns 5 to 10 days post-administration for a measurement and reassessment. The workbooks evaluate the radiation field strength external to the patient and indicate restrictions based upon selected dose limits. To assist in suggesting contamination precautions, the workbooks also evaluate the radioactivity present within the patient and the estimated discharge into the environment as a function of time.
Conclusions

The workbooks, a user’s manual, and a document detailing the mathematics involved are available free of charge by email request sent to the author.

**Key words:** radiopharmaceutical therapy, $^{131}$I therapy, radiation protection, radiation safety, mathematical modeling
Introduction

The radiation safety counseling of $^{131}$I therapy patients and their families is provided by facilities that offer this therapy. Published guidelines to facilitate this process are necessarily generic in nature. The guideline provided by the American Thyroid Association, for example, assumes that the thyroid remnant of an ablation patient has an $^{131}$I uptake of 2%, while the intact thyroid of a hyperthyroid patient has an uptake of 50% (1). To avoid imposing excessive restrictions upon a large number of families, these are reasonable assumptions. However, if the surgeon were to have reported an unusually large thyroid remnant, then the 2% assumption may not be appropriate. Also, it is not unusual to encounter a hyperthyroid patient whose uptake significantly exceeds 50%. To deal with these less common situations, more severe restrictions are needed, but we wish them to be no more oppressive than necessary.

An Excel spreadsheet that generates customized restrictions for a patient and his or her family has been reported (2). Upon entering a few parameters, individual restrictions are then automatically computed for adult family members, the sleeping partner, held children, and co-workers. Use of such a spreadsheet transforms individualized dose computations from a dreaded into a trivial process, and a printout of the sheet can be filed as documentation for review by regulators during inspections. In the work presented below, this spreadsheet has been expanded. By modeling the patient’s extra-thyroidal compartment as a volume source, its influence on the radiation field strength external to the patient vs that of the thyroid point source can be realistically followed over the course of therapy. A separate model is developed to estimate the radionuclide activity as a function of time that remains in the extra-thyroidal compartment as well as the activity released at each urination. Finally, worksheets are added to permit reassessment, a few days post administration, of particularly oppressive restrictions on a patient’s activities,
and then to develop a strategy to resolve the conflict between patient needs and acceptable dose limits to family members.

These worksheets are partitioned into four Excel workbooks. Workbook #1, labeled “Ablation—Thyrogen Preparation”, is intended for use with $^{131}$I ablation patients when the patient’s overall TSH level has been enhanced by administration of exogenous human TSH (Thyrogen). The therapeutic goal for these post-thyroidectomy patients, of course, is to destroy the thyroid remnant and potential metastatic disease. Workbook #2, labeled “Ablation—Prepared by Deprivation”, is for use when the ablation patient’s TSH has been enhanced by thyroid hormone withdrawal. Workbook #3, labeled “Hyperthyroid—Initial Calculations”, is for use, initially, with all hyperthyroid patients. Workbook #4, labeled “Hyperthyroid—Reassessment by Measurement”, is for use in the unusual situation that a hyperthyroid patient finds the restrictions of Workbook #3 to be oppressive, and returns 5 to 10 days after $^{131}$I administration to have the strength of the radiation field measured so that restrictions may be reassessed and a strategy developed to satisfy patient needs while providing adequate radiation protection of children in the family.

**Methods and Materials**

Two simple models are used to provide estimates of $^{131}$I behavior in the therapy patient. A geometric representation of the first, called the parallel model, is given in figure 1. There are two compartments, corresponding to the extra-thyroidal space and the thyroid. Because the number of compartments is two, a biphasic retention curve is produced as illustrated in figure 2. The parallel model does not provide a realistic representation of the isotope’s flow pattern within the patient; its purpose is only to create a biphasic retention curve which, for a critical few days following administration, is found to
satisfactorily represent the temporal behavior of the external radiation field strength about the patient.

To provide a somewhat more realistic representation of the isotope’s flow pattern within the patient, the second model, the serial model, is introduced as illustrated in figure 3. Here, rather than the thyroidal compartment excreting the isotope directly, as depicted in the parallel model, the isotope migrates into the extra-thyroidal space and from there into the urinary bladder. The advantage of the parallel model is that mathematically it is much simpler than the serial model, yet it satisfactorily represents the temporal behavior of the external radiation field about the patient, while the serial model is a more realistic depiction of the isotope’s migration within the patient. Table 1 provides a key to the notations used during the mathematical developments of the two models that follow.

Parallel Model
For the parallel model, figure 1, the basic mathematics for both compartments is identical, although the values of the constants $F$ and $\lambda_{bio}$ differ. For each of the two compartments there is no entrance pathway for the isotope and there are two exit pathways, by transition to the excretion compartment and by radioactive decay. The differential equation for the $i^{th}$ compartment is then

$$\frac{dN_i(t)}{dt} = -\lambda_{bio}N_i(t) - \lambda_{phys}N_i(t).$$

With the initial condition, $N_i(0) = F_iN_0$, the solution to this equation is

$$N_i = N_0F_ie^{-\lambda_{eff}t}.$$  

The total isotope retention as a function of time (figure 2) is then

$$N(t) = N_0\{F_1e^{-\lambda_{eff}t} + F_2e^{-\lambda_{eff}t}\}.$$
The air kerma rate from the patient is the product of four factors: the activity of the radionuclide (given by the equation above), its air kerma rate constant, the fraction of radioactive emissions that is not absorbed within the patient, and a distance factor that is dependent upon the geometry of the radiation source and the distance from the patient to the dose point. Therefore, in units of mGy/h, the air kerma rate from the thyroid, modeled as a point source, is

\[
\dot{K}_{a2}(t) = \frac{\Gamma N_0 F_2 B_2 e^{-\lambda_{eff} t}}{(d_2 + \Delta d_2)^2}.
\]

To simulate the extra-thyroidal volume source, it was modeled as a cylinder of water within which was placed 37 line sources parallel with the cylinder’s axis. Each line source had an initial activity \(1/37\)th of that taken up by the extra-thyroidal compartment, and the activity was uniformly distributed along the source. To provide a uniform distribution of sources throughout the cylinder, each source was centered within contiguous areas, each area equal to \(1/37\)th of the cylinder’s cross-section. The composite radiation field strength external to the cylinder including self-absorption for each line source was computed as

\[
\dot{K}_{a1}(t) = \frac{\Gamma N_0 F_1 e^{-\lambda_{eff} t}}{L} \sum_{i=1}^{37} \int_{-L/2}^{L/2} e^{-\frac{\ln(2)\Delta g_i}{\ln L} \sqrt{g_i^2 + g_i^2}} \frac{g_i^2}{l^2 + g_i^2} \, dl,
\]

where \(g_i\) is the perpendicular distance from the \(i\)th line source to the dose point, \(\Delta g_i\) is the perpendicular distance from the \(i\)th line source in the direction of the dose point to the intersection with the cylinder wall, and when \(\Delta g_i\) is multiplied by the ratio \(\sqrt{l^2 + g_i^2} / g_i\) the product is the diagonal distance between the line source element, \(dl\), and the dose-point ray’s intersection with the cylinder’s wall. The numerator of the integrand is then the cylinder’s transmission of radiation from this line source element. The integration for each of the 37 line sources was performed numerically using Excel. The composite air kerma rate at the dose point, i.e., at the position of the bystander, in units of mGy/day is
\[ K_a(t) = 24\left[K_{a1}(t) + K_{a2}(t)\right], \]

where 24 is the conversion, h/day. Recognizing that air kerma is air kerma rate times time, from the previous three equations the total air kerma due to emissions of the radioisotope retained by the patient that is received by a bystander who is present a fraction \( T_{oc} \) of the time after a start time \( t_s \) is

\[
K_a(t_s) = 24T_{oc} \Gamma N_0 \left\{ \frac{F_1}{L} \int_{t_s}^{\infty} e^{-\lambda_{eff1} t} dt \right\} + \sum_{i=1}^{37} \int_{-L/2}^{L/2} e^{-\frac{ln(2)}{NVL} \frac{\Delta g_i}{g_i} \frac{[l^2 + g_i^2]}{l^2 + g_i^2}} \frac{dL}{l^2 + g_i^2} + \frac{F_2 B_2}{(d_z + \Delta d_z)^2} \int_{t_s}^{\infty} e^{-\lambda_{eff2} t} dt \right\}.
\]

Making the substitution \( \lambda = \frac{ln(2)}{T_{1/2}} \) and completing the integration with respect to time, this total air kerma (mGy) becomes

\[
K_a(t_s) = 34.62T_{oc} \Gamma N_0 \left\{ \frac{T_{1 eff1}}{L} e^{-\frac{ln(2)}{T_{1 eff1} t_s}} \right\} + \sum_{i=1}^{37} \int_{-L/2}^{L/2} e^{-\frac{ln(2)}{NVL} \frac{\Delta g_i}{g_i} \frac{[l^2 + g_i^2]}{l^2 + g_i^2}} \frac{dL}{l^2 + g_i^2} + \frac{T_{1 eff2} F_2 B_2}{(d_z + \Delta d_z)^2} e^{-\frac{ln(2)}{T_{1 eff2} t_s}}, \right\}, \tag{1}
\]

where \( 34.6247 = \frac{24}{ln(2)} \). The transmission of radiation from the thyroid, \( B_2 \), is simply

\[ B_2 = e^{-\frac{ln(2)}{NVL} \Delta d_z}, \]

where \( \Delta d_z \) is the water equivalent thickness of tissue that the 364 keV photon is assumed to traverse as it escapes from the thyroid (the author uses the estimate \( \Delta d_z = 1.5 \) cm, i.e., 0.015 m).

To investigate the effect of patient size on the radiation field strength, the author computed the air kerma to bystanders using for the extra-thyroidal source a variety of cylinder diameters. Note that as the cylinder size changed the 37 line sources
redistributed automatically in order to maintain a uniform dispersal of sources throughout the entire volume. As expected, the radiation field strength exposing a bystander decreases as the size of the patient increases, due partly to increasing the volume through which the source is distributed, but primarily to an increase in self-absorption. During this investigation, it was a fortuitous discovery that, by replacing the 37 line sources with a single line source of their combined activity, the computed radiation field strength external to the patient from this single source can duplicate that of the 37 sources to within a couple percent over the distance range from the patient of 0.1 to 3 m.

Geometrically this single line source is parallel to the cylinder’s axis, it is in the plane determined by this axis and the dose point, and it is the length of the cylinder. For the resulting computed external field strength to replicate faithfully that of the distributed 37 sources, the single line source must be at a specific depth that depends upon the cylinder’s diameter. It may be important to note that the computed self-absorption of the single line source at this specific depth differs somewhat from that of the cylinder with the 37 line sources; however, this is irrelevant in the sense that the external field strength is nevertheless truthful. Thus Equation 1 is replaced with

\[
K_d(t_s) = 34.6247T_{oc} \Gamma N_0 \left[ T_{\frac{t}{2\text{eff}1}} F_1 L e^{-\left(\frac{\ln(z)}{T_{\frac{t}{2\text{eff}1}} t_1}\right)} - \int_{-L/2}^{L/2} e^{-\left(\frac{\ln(z)\Delta d_1}{N t_1}\right)} \frac{\left[\frac{t_2}{T_{\frac{t}{2\text{eff}1}}} \right]}{t^2 + (d_1 + \Delta d_1)^2} \, dl \right] + T_{\frac{t}{2\text{eff}2}} F_2 e^{-\left(\frac{\ln(z)\Delta d_2}{N t_2}\right)} e^{-\left(\frac{\ln(z) t_3}{T_{\frac{t}{2\text{eff}2}}}\right)},
\]

where in the integrand \(\Delta d_1\) is the specific depth of the single line source corresponding to the selected cylinder (patient) diameter, and \(d_1 + \Delta d_1\) is the total distance from the line source to the dose point, i.e., the bystander. The first term in the curly brackets
represents the extra-thyroidal (cylindrical source) component and the second term the thyroidal (point source) component of the computation. Equation 2 is used to compute all doses to bystanders from patient emissions provided by the workbooks.

Serial Model

The thyroid compartment (compartment 2), as for the parallel model, has no entrance pathway for the isotope and two exit pathways, by radioactive decay and by transition to the extra-thyroidal compartment (see figure 3). The differential equation is then

$$\frac{dN_2(t)}{dt} = -\lambda_{\text{bio2}}N_2(t) - \lambda_{\text{phys}}N_2(t),$$

and with the initial condition that $N_2(0) = N_0F_2$, the solution is

$$N_2(t) = N_0F_2e^{-\left(\lambda_{\text{phys}} + \lambda_{\text{bio2}}\right)t}. \quad (3)$$

Equation 3 is the basis for the patient’s thyroid activities computed by the serial model.

The extra-thyroidal space (compartment 1) has one entrance pathway, by transition from the thyroid (compartment 2), and two exit pathways, by radioactive decay and by transition to the excretion/secretion compartment (represented as “Bladder” in figure 3). The differential equation is then

$$\frac{dN_1(t)}{dt} = \lambda_{\text{bio2}}N_2(t) - \lambda_{\text{bio1}}N_1(t) - \lambda_{\text{phys}}N_1(t),$$

where $N_2(t)$ is given in Equation 3. With the initial condition, $N_1(0) = F_1N_0$, as shown in reference (3) the solution is

$$N_1(t) = \left\{ \frac{\lambda_{\text{bio2}}F_2}{\lambda_{\text{bio1}} - \lambda_{\text{bio2}}} e^{-\lambda_{\text{bio2}}t} + \left[ F_1 - \frac{\lambda_{\text{bio2}}F_2}{\lambda_{\text{bio1}} - \lambda_{\text{bio2}}} \right] e^{-\lambda_{\text{bio1}}t} \right\} N_0 e^{-\lambda_{\text{phys}}t}. \quad (4)$$

Equation 4 is the basis for the patient’s extra-thyroidal space activities computed by the serial model.
The excretion/secretion compartment (compartment 3) has one entrance pathway, by transition from the extra-thyroidal space (compartment 1), and one exit pathway, by radioactive decay (see figure 3). The differential equation is then

$$\frac{dN_3(t)}{dt} = \lambda_{bio1} N_1(t) - \lambda_{phys} N_3(t),$$

where $N_1(t)$ is given in Equation 4. With the initial condition that at some arbitrary time $t=T$, $N_3(T) = 0$, as shown in reference (3) the solution is

$$N_{3T}(t) = \left( \frac{\lambda_{bio1}F_2}{\lambda_{bio1}-\lambda_{bio2}} e^{-\lambda_{bio2}T} - e^{-\lambda_{bio2}t} \right) + \left[ F_1 - \frac{\lambda_{bio2}F_2}{\lambda_{bio1}-\lambda_{bio2}} \right] e^{-\lambda_{bio1}T} - \left( e^{-\lambda_{bio1}t} \right) N_0 e^{-\lambda_{phys}t}. \quad (5)$$

With the approximation that all excretions accumulate as urine in the bladder, Equation 5 provides the isotope’s activity to be urinated at time $t$, if the previous urination had occurred at time $T$. Letting $T=0$ and recognizing that $F_1 + F_2 = 1$, Equation 5 becomes

$$N_3(t) = \left[ 1 - \frac{\lambda_{bio1}F_2}{\lambda_{bio1}-\lambda_{bio2}} e^{-\lambda_{bio2}t} - \left( F_1 - \frac{\lambda_{bio2}F_2}{\lambda_{bio1}-\lambda_{bio2}} \right) e^{-\lambda_{bio1}t} \right] N_0 e^{-\lambda_{phys}t}. \quad (6)$$

Equation 6 provides the total isotope activity excreted since administration that has not yet decayed.

Results

The four Excel workbooks are listed and briefly described in the introduction. The first three workbooks each have two worksheets, labeled EXT and INT. The EXT sheet evaluates the radiation field strength external to the patient using Equation 2 (parallel model). The upper section of this sheet that contains the cells used to input values needed for Equation 2 is shown in figure 4. Values for $T_{ac}$ (cells J5 through J8), $N_0$ (cell O2), $d_1$ (G5 through G8), $d_2$ (H5 through H8), $\Delta d_2$ (O5), $T_{eff1}$ and $T_{eff2}$ (N8 and O8), $F_1$ and $F_2$ (N7 and O7), are input, and resulting air kerma, assumed equivalent to effective dose, then appear in rows 15 and below (see Table 1 for symbol definitions). Notice that
\( \Delta d_1 \), the depth of the extra-thyroidal line source in cell N5, is not input (no yellow fill), because this parameter is computed based upon the patient diameter entered by the user in H2. As an example of the flexibility afforded by the workbooks, in cell G8 notice that for the thyroidal point source the patient-to-held-child distance is 30 cm (0.3 m), but, in cell H8, that for the extra-thyroidal line source this distance is input as only 10 cm (0.1 m). The rationale for this apparent discrepancy is that the infant, being bottle fed by the patient, is in direct contact with the patient’s abdomen/thorax, but at a greater distance from the thyroid remnant.

The INT sheet, not shown, evaluates and displays, as a function of time, the radionuclide activities in the patient’s thyroid, extra-thyroid space, and bladder using Equations 3 through 6 (serial model). It also estimates the amount of radionuclide discharged at each urination. This information may have application when the risk of contamination or nuclide transfer, e.g., to an infant, may be a concern.

The two workbooks for ablation patients (for patients prepared by thyrogen vs. by hormone deprivation) are separated to call attention to the fact that kidney function is known to be profoundly influenced by thyroid status (4). Because removal of \(^{131}\)I by the kidneys from the extra-thyroidal compartment is substantially delayed for hypothyroid patients, the default value of \(T_{1/2}^{1}\text{eff}\) for patients prepared by deprivation is 16 hours, while that for those prepared using thyrogen, who are assumed to be euthyroid, is 8 hours. Another observation of interest in this context is that there is indication that the thyroid of patients prepared by thyrogen administration may retain the iodine somewhat longer than those prepared by deprivation (5).
One anticipates that the uptake values, \( F_1 \) and \( F_2 \), and the nuclear decay, \( \lambda_{phys} \), should be identical in the parallel and serial models; however, due to the obvious kinetic differences between the two models as depicted in figures 1 and 3, values of the biological rate constants, \( \lambda_{bio1} \) and \( \lambda_{bio2} \), for the serial model are expected to differ somewhat from the values employed in the parallel model. Because we set these rate constants of the parallel model to simulate what we expect to be the true temporal behavior of the radiation field strength external to the patient, we use this temporal behavior to determine the biological rate constants for the serial model. Specifically, we first generate the biphasic time profile (see figure 2 for such a profile) of the isotope retention using each model. Then, in iterative fashion using the least square minimization technique, the serial model’s \( \lambda_{bio1} \) and \( \lambda_{bio2} \) are determined so that the biphasic time profiles of the two model’s match. A detailed description of this process is available (6).

The third of the four workbooks, labeled “Hyperthyroid—Initial”, consists of an \( \text{EXT} \) and an \( \text{INT} \) sheet as described above. The intact thyroids of these patients often have unusually high iodine uptake. As a consequence, even though relatively low activities of \(^{131}\text{I} \) are administered to these patients, external radiation field strengths can be high enough and persistent enough to require severe restrictions regarding the patient’s associations with children in the family, depending upon the dose constraint that is employed for children. For these patients a recent thyroid radioiodine uptake study is recommended (7), but even if a thyroid uptake is available, there are large uncertainties regarding the value of \( \lambda_{bio2} \), i.e., the rate that \(^{131}\text{I} \) leaves the thyroid by biological transition. In a group of 127 hyperthyroid patients, both the thyroid’s biological half-life for radioiodine release and the uptake of radioiodine were studied (8). With the biological half-life plotted against the uptake fraction, there appears to be
an inverse relationship between uptake and half-life, but the data are so scattered
(the biological half-life varies from nearly zero to about 75 days) that no specific
half-life can be reasonably associated with an uptake value. The author drew a
straight line through the data that placed approximately 90% of the half-lives below
and 10% above the line. In the EXT sheet of Workbook #3 this line, and the
measured uptake value, are used to determine the biological half-life, $T_{bio2}^1$, for
release of iodine from the thyroid of a hyperthyroid therapy patient. Thus we
anticipate that restrictions based upon this workbook that are placed upon a
hyperthyroid therapy patient's interaction with children will be overly restrictive
90% of the time.

To deal with this issue, Workbook #4 was prepared, labeled “Hyperthyroid—
Reassessment by Measurement”. It is recommended that this workbook be
employed only in the unusual situation that the patient views the restrictions as
oppressive. When this workbook is employed the patient returns for reassessment
5 to 10 days post-administration, the time to be selected when it is believed that the
$^{131}$I pool in the extra-thyroidal space is sufficiently depleted that essentially all
residual $^{131}$I activity is localized in the thyroid (the INT worksheet may assist here).
The EXT sheet of this fourth workbook has been extended to include the following
considerations. The radiation field strength 1 m from the patient’s thyroid in units
of air kerma (mGy/h) is

$$\hat{K}_a(t) = N_0 \Gamma F_2 B_2 \left( \frac{t}{T_{bio2}^1} \right),$$

where $\hat{K}_a(t)$ is determined by a careful µSv/h measurement at 1 m from the patient’s
thyroid taken $t$ days (5 to 10 days) after administration of the $^{131}$I. Our primary mission
here is to determine from this measurement the actual value of $T_{1/2}^{\text{eff}2}$ for this patient, thus removing the conservative aspect of the earlier estimate (see the previous paragraph). Solving the equation above for this parameter

$$T_{1/2}^{\text{eff}2} = \frac{t\ln 2}{-\ln \left( \frac{K_a(t)}{N_0 F_2 B_2} \right)}.$$  \hspace{1cm} (7)

If Equation 7 yields a value for the half-life of 3 days or more, this value is accepted and the EXT worksheet is recomputed providing updated restrictions. When Equation 7 yields a half-life of less than 3 days (the author considers three days to be a reasonable minimum value for the effective half-life for radioiodine release), a thyroid uptake value of less than $F_2$ is suspected. Solving the equation for $F_2$ while maintaining the half-life at 3 days,

$$F_2 = \frac{\dot{K}_a(t)}{N_0 F_2 B_2 2^{-t/3.0}}.$$  \hspace{1cm} (8)

In summary, the EXT sheet of Workbook #4 executes a protocol to determine a value for the rate at which $^{131}$I leaves this patient’s thyroid and, when needed, a new value for the thyroid uptake. The protocol is based upon a careful measurement of the air kerma rate 1 m from the patient’s thyroid. Once this parameter is input along with the corresponding post-administration time, the resulting actual values of $T_{1/2}^{\text{eff}2}$, and $F_2$ for this patient are computed and used in Equation 2 to reassess the initial restrictions placed upon the patient’s interaction with children in the family.

Occasionally, to support maternal bonding, there is an urgency to gradually integrate the patient into the care of an infant as effectively as possible. To deal with this a second worksheet to Workbook #4, labeled RATE, is introduced. The user tags each of four columns of the RATE sheet with patient-to-infant distances. For example the author usually tags the first column for bottle feeding (0.1 m for the extra-thyroidal cylindrical
source and 0.3 m for the thyroidal point source), the second column for bathing and diaper changing (0.5 m), the third column for social interactions (1 m), and the fourth column for more remote duties the patient may have (3 m). Dose rates are then computed and displayed in each column for the corresponding user-specified distance. The dose rates have been computed using equation (2) and averaged over each 24-hour interval beginning with the 0th day and continuing through day 69. Next, the user and/or the patient enter a trial number of hours for each of the four patient-to-infant distances and for each 24-hour interval. The workbook displays the doses that accumulate as a consequence. When the running total dose advances too rapidly, the user/patient returns to early times, and probably to short distances, to reduce the number of hours that had been previously tried. By repeatedly reconfiguring the trial-number of hours the patient develops a strategy that meets the selected dose constraint for the infant while most effectively addressing the perceived urgency.

**Discussion**

A reasonably comprehensive Excel-based system to evaluate and assure the radiation protection of those in the vicinity of an ¹³¹I therapy patient has been presented. The author is aware of difficulties associated with some previous systems developed for this purpose, and in the course of developing the current product has attempted to address some of these. A frequent concern is that these methods tend to be overly conservative (9,10). In the approach presented here the radiation emanating from the thyroid is modeled as a simple point source attenuated by a thickness of water-equivalent material specified by the user, while the extra-thyroidal source is modeled as a cylindrical volume 1 m long with nearly uniform source intensity throughout, and incorporating self-absorption that corresponds to 364 keV photons in water. The user selects a diameter for the cylinder that can vary from 15 to 40 cm. The 15 cm diameter source corresponds to a
slender patient of youthful build while the 40 cm source corresponds to a very large patient of possibly rather obese construction, and, as a consequence, will have greater self-absorption. The strength of these two sources is initially input as a total administered radionuclide activity that is partitioned by two corresponding uptake fractions that add to 1. The kinetic behavior of the radionuclide following administration is discussed in detail above. For hyperthyroid therapy cases there is a large uncertainty regarding the rate at which the radionuclide leaves the thyroid by biological transition. Occasionally the initial conservative estimate for this parameter may lead to oppressive restrictions. In this case the system presented here has the patient return 5 to 10 days after administration to have this biological rate constant measured. This is a simple process involving a single measurement with a radiation survey meter as discussed above. Although the corrected (measured) rate of escape from the thyroid often results in relaxation of the restrictions, the frequency of the relaxation being small is such that the initial estimates do not appear to be excessively conservative. It is usually dose limits to children in the family that prompt the patient’s return for this measurement, and the parents seem pleased with the assurance provided by the measurement that the children are being well protected. They are thankful, of course, whenever the restrictions are relaxed.

The author’s selection of 1 m for the length of the cylindrical source used to model the extra-thyroidal compartment is somewhat arbitrary. Most individuals are closer to 1.7 m in length, and much of the extra-thyroidal space is circulating blood which is distributed over the entire body’s length. However, a shorter length for the cylinder is appropriate because the patient will be sitting much of the time, will be not completely straight while sleeping, and the radiation source strength may not be uniform along the entire body length. In estimating the effective length of the cylindrical source, assumed to be uniform, the author wishes any error to be in the conservative direction, so 1 m is
selected. Note that modeling the extra-thyroidal source as a cylindrical volume rather than as a point is a dramatic step toward realism, while extending the length of the cylinder makes relatively little difference.

There is concern that Equation 2 may underestimate the dose to a child held by the patient. The child, being in direct contact with the abdomen/thorax of the patient, which is modelled as a cylindrical volume source, is subject to a population of multiply scattered photons that differs from that of bystanders who are physically separated from the cylindrical source. To assess this concern the configuration of the 37 line sources within the cylinder was held constant as the cylinder was expanded by an annular ring of water (with no radioactivity) 20 cm thick. The air kerma midway within the annular ring, representing the dose point for the held child, was computed. Because the position of the child is now within the expanded cylinder, self-absorption was computed using buildup factors and the linear attenuation coefficient (reference (11) pages 286-7 and 223 respectively) rather than using the broad-beam HVL as before. The buildup factors were either computed by or verified by a Monte Carlo code to assure that scattered photons were incorporated accurately. For this artificial worst case scenario the held child doses increased by only 6.4% for the smallest cylinder and 13.4% for the largest (diameters of 15 and 40 cm, respectively). In the context of all the uncertainties inherent in the family counseling process, these variations are trivial and can be ignored.

We address concerns that assuming air kerma to be equivalent to effective dose may be excessively conservative. A Joint Task Group of the ICRP and the ICRU has provided effective-dose to air-kerma conversion factors as a function of bystander age and geometrical orientation with respect to the beam (12). For the 364-keV photon and for children 0 to 5 years old, effective dose is 80 to 90% of the air kerma for isotropic beam
incidence (equal beam strength from all directions). For the adult bystander a more appropriate beam orientation might be for the beam to be orthogonal to the adult’s longitudinal axis, but then rotate about this axis with no preferred direction. For this conformation of 364-keV photons incident upon the adult bystander the effective dose is about 82% of the air kerma. In both cases the assumption that air kerma is equivalent to effective dose is conservative by only 10 to 20%. However, if these photons were incident on the anterior of the child or the adult, the effective dose would be 110% of the air kerma. The latter configuration would be appropriate for a child being held by the patient, and possibly for the patient’s sleeping partner. Thus assuming equivalence of effective dose and air kerma is not overly conservative, and in some cases it may fail to be sufficiently conservative.

As stated in the introduction, the worksheet that is central to the system presented here, the EXT sheet, is configured after one described previously (2). Although the two spreadsheets are similar in appearance, substantial differences between the two approaches exist. First, for the strength of the radiation field the method of reference (2) uses exposure measured at two distances from the patient (their example uses 0.3 and 1.0 m) immediately following administration of the radionuclide. This measurement incorporates the self-absorption present before the radionuclide has distributed. This initial source strength is then decayed by the effective half-lives corresponding to the biphasic retention. The method presented in this paper computes the source strength from the radionuclide activity remaining in the thyroid and the extra-thyroid space, reduced in intensity by the self-absorption computed separately for each of these compartments. The resulting source strengths of these two methods will differ substantially for hyperthyroid patients that have a large thyroidal iodine uptake because the self-absorption of radiation emanating from the thyroid is much less than that
emanating from the patient immediately following administration. Second, the method of reference (2) is designed to be used when a therapeutic quantity of any radiopharmaceutical is employed, while this paper is focused upon the therapeutic use of $^{131}$I. Third, reference (2) suggests that uptake fraction and effective half-life for the thyroidal and extra-thyroidal compartments be obtained from either relevant scientific literature, or by actual measurement using a pretherapy tracer administration of the radiopharmaceutical to be employed. The institution that originated the method of reference (2) (Memorial Sloan Kettering, NY) prefers use of the pretherapy tracer, though this is not commonly employed in most practices. The method of this paper employs a conservative estimate of effective half-life derived from data of the scientific literature combined with a measured thyroidal uptake as explained above. In addition to these 3 differences, the method of reference (2) has been expanded: first, to recognize the importance of reduced kidney function for the hypothyroid ablation patient relative to that of the patient prepared using thyrogen; second, to estimate the amount of radionuclide activity expelled at each urination, third; to provide updated restrictions based upon a single measurement of radiation field strength 5 to 10 days following administration whenever the family finds the original restrictions to be oppressive; and fourth, to easily devise a personalized strategy to permit integration of the patient into the care of an infant while complying with dose limits selected for the infant.

Both the EXT worksheet and the spreadsheet presented in reference (2) ignore dose to a bystander due to ingestion of radioactive contamination. When the bystander is a child (or pregnant woman) with a dose limit of only 1 mSv, and the family cannot sequester the child for a few days into a region of the household separated from that occupied by an ablation patient, this could become an issue. In such a situation, by ignoring internal dose the potential exists for the spreadsheets to deceive the counselor into believing that
compliance is achieved, when in fact it may not be. (The author has observed that infants’ mouths, when teething, are attracted by the cool, smooth texture of a toilet’s porcelain.) The INT sheet in the workbooks is intended, in part, to provide an alert regarding contamination potential.

Conclusion

A system of four Excel workbooks has been developed that assists the radiation safety counselor to implement individualized procedures regarding radiation protection of the $^{131}$I patient’s family members, and others. The system avoids overly conservative assumptions, while permitting the counselor to select dose limits for each individual as deemed appropriate. Demands on the part of the counselor are minimal.

The system is available free of charge via an email request to the author. This offer is basically “as is”, though the author will attempt to respond to requests and provide error corrections. The system will be forwarded to requesters by return email to include the four Excel workbooks, a User’s Manual, and a document detailing the mathematics contained in the workbooks.
Disclosures: the author has nothing to disclose.

Acknowledgements: The greatest contribution to the development of this product came from the patients and their families referred to me for radiation safety counseling. The sharing with me of their experience, circumstances, and concerns provided the direction and the motivation for developing the product. Richard T. Nelson, MD and Kathleen Figaro, MD are acknowledged for making the patient referrals and for their general support during project development. The author particularly appreciates the efforts of Mark Madsen, PhD for reviewing this work in an intermediate stage of development and for his comments and encouragement to share the work with the nuclear medicine community. The author acknowledges his wife, Karen Beetham, PhD, who has corrected and improved multiple drafts of this paper, the User’s Manual (6), and the document detailing the mathematics contained in the workbooks (3).
References


3. Steward PG. Mathematical basis for the radiopharmaceutical workbooks. Unpublished (Available from the author by E-mail request).


6. Steward PG. Workbooks for counseling $^{131}$I therapy patients and families—a user’s manual. Unpublished (Available from the author by E-mail request).


FIGURE 1 A geometric representation of the parallel model. The fraction of the administered radionuclide that initially resides in the extra-thyroidal space and the remaining fraction taken up by the thyroid are represented by $F_1$ and $F_2$, respectively. The radionuclide in each of these compartments biologically transfers directly into the excretion compartment with rate constants $\lambda_{\text{bio}1}$ and $\lambda_{\text{bio}2}$. Although this flow pattern is biologically unrealistic, it provides the typical biphasic retention profile, and it has the advantage of being mathematically simple. The radionuclide content of the extra-thyroidal space, modelled as a uniformly distributed cylindrical volume source, and the content of the thyroid, modelled as a point source, determine the radiation field strength in the vicinity of the patient.
FIGURE 2 A biphasic retention function generated by the parallel model using parameters that would be suitable for a thyrogen-prepared $^{131}$I ablation patient. For phase 1 (extra-thyroidal space) the initial uptake is 97% and the effective half-life is 8 hours. For phase 2 (thyroid) the initial uptake is 3% and the effective half-life is 5 days.
FIGURE 3 A geometric representation of the serial model. The radionuclide in compartment 2 (thyroid) biologically transfers into compartment 1 (extra-thyroidal space), and that in compartment 1 transfers to the bladder, which then periodically empties into the environment. This model is used to estimate the radioactive content of each compartment and of periodic urinations as a function of time.
FIGURE 4  A small section of the **EXT** worksheet of Workbook #2. The cells that are unprotected and thus available to the user for input have yellow fill. Cells corresponding to forbidden doses have magenta fill, and those with permitted doses are green.
### TABLE 1

**Mathematical Notations and Definitions**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_2$</td>
<td>the transmission factor for radiation emanating from compartment 2 (thyroid)</td>
</tr>
<tr>
<td>$d_1$</td>
<td>the distance (meters) to a bystander from the patient’s skin at the extra-thyroidal source</td>
</tr>
<tr>
<td>$d_2$</td>
<td>the distance (meters) to a bystander from the patient’s skin at the thyroid</td>
</tr>
<tr>
<td>$\Delta d_1$</td>
<td>the depth in the patient (meters) of the extra-thyroidal line source</td>
</tr>
<tr>
<td>$\Delta d_2$</td>
<td>the depth in the patient (meters) of the thyroidal point source</td>
</tr>
<tr>
<td>$F_1 = \frac{N_1(0)}{N_0}$</td>
<td>the fraction of administered isotope initially partitioned into compartment 1</td>
</tr>
<tr>
<td>$F_2 = \frac{N_2(0)}{N_0}$</td>
<td>the fraction of administered isotope initially partitioned into compartment 2</td>
</tr>
<tr>
<td>$g_i$</td>
<td>the perpendicular distance from the cylinder’s $i^{th}$ line source to the dose point</td>
</tr>
<tr>
<td>$\Delta g_i$</td>
<td>the perpendicular distance from the cylinder’s $i^{th}$ line source to the cylinder’s surface in the direction of the dose point</td>
</tr>
<tr>
<td>$HV L$</td>
<td>The broad-beam half-value-layer in water (meters) of $^{131}$Iodine’s 364-keV photon (the value 0.1 m is used, reference (11) page 305)</td>
</tr>
<tr>
<td>$K_a$</td>
<td>the air kerma rate at the position of a bystander resulting from patient emanations</td>
</tr>
<tr>
<td>$K_a(t_s)$</td>
<td>the total air kerma at the position of a bystander beginning at a start time $t_s$</td>
</tr>
<tr>
<td>$l$</td>
<td>variable of integration, distance from midpoint of a line source to line source element $dl$</td>
</tr>
<tr>
<td>$L$</td>
<td>the length (meters) of the line source</td>
</tr>
<tr>
<td>$N_1(t)$</td>
<td>the amount of isotope remaining in metabolic compartment 1 (extra-thyroid) at time $t$</td>
</tr>
<tr>
<td>$N_2(t)$</td>
<td>the amount of isotope remaining in metabolic compartment 2 (thyroid) at time $t$</td>
</tr>
<tr>
<td>$N_{3T}(t)$</td>
<td>the amount of isotope in the bladder at time $t$ when the previous urination was at time $T$</td>
</tr>
<tr>
<td>$N(t)$ = $N_1(t) + N_2(t)$</td>
<td>the amount of isotope remaining in the patient at time $t$ (parallel model)</td>
</tr>
<tr>
<td>$N_0 = N_1(0) + N_2(0)$</td>
<td>the amount of isotope initially administered</td>
</tr>
<tr>
<td>$T_{1eff}$</td>
<td>the effective half-life for the radioisotope’s leaving metabolic compartment 1</td>
</tr>
<tr>
<td>$T_{2eff}$</td>
<td>the effective half-life for the radioisotope’s leaving metabolic compartment 2</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>the air kerma rate constant (mGy/h/MBq at 1 m) for the radioisotope (the value 5.1613E-2 is used for $^{131}$I)</td>
</tr>
<tr>
<td>$\lambda_{phys}$</td>
<td>the rate constant for the radioisotope’s leaving a metabolic compartment by nuclear decay</td>
</tr>
<tr>
<td>$\lambda_{bio1}$</td>
<td>the rate-constant for the isotope’s leaving metabolic compartment 1 by biological processes</td>
</tr>
<tr>
<td>$\lambda_{bio2}$</td>
<td>the rate-constant for the isotope’s leaving metabolic compartment 2 by biological processes</td>
</tr>
<tr>
<td>Notation</td>
<td>Definition</td>
</tr>
<tr>
<td>----------</td>
<td>------------</td>
</tr>
<tr>
<td>$\lambda_{\text{eff}1} = \lambda_{\text{bio}1} + \lambda_{\text{phys}}$</td>
<td>the effective rate constant for the isotope leaving metabolic compartment 1</td>
</tr>
<tr>
<td>$\lambda_{\text{eff}2} = \lambda_{\text{bio}2} + \lambda_{\text{phys}}$</td>
<td>the effective rate constant for the isotope leaving metabolic compartment 2</td>
</tr>
<tr>
<td>$\lambda = \frac{\ln(2)}{T_{1/2}}$</td>
<td>the relationship between rate constant and half-life, where $\ln(2)$ is the natural log of 2</td>
</tr>
</tbody>
</table>