Letters to the Editor

CHOOSING A GOODNESS-OF-FIT TEST

I have been reading the article by John Prince in the March 1977 issue, entitled "Goodness-of-Fit Tests for Describing the Statistical Behavior of Nuclear Counting Equipment." The author has gathered together a number of useful statistical tests, but I feel his paper is significantly less valuable than it might have been if he had discussed the relative strengths of the tests described. He mentions ease of computation, which is of course a relevant factor, but it is certainly not the only one. A comparison of the tests for this particular application is outlined below.

Three of the four tests described in the article—Lexis' divergence coefficient, Q^2, the Reliability Factor (RF), and the chi-square test, χ^2—are essentially identical since simple one-to-one relationships exist between the three test statistics involving only the sample size n:

$$Q^2 = \chi^2$$

and

$$RF = [\chi^2/(n-1)]^{1/2}.$$

Consequently, if any one of these tests gives a significantly high or low result from a particular set of data the other two will do likewise. The critical points for all three tests are derived from those of the χ^2 distribution so that it is the last of these tests, the chi-square dispersion test as it is commonly called, which is most widely used. Tables of percentage points of the χ^2 distribution can be found in almost any elementary statistics textbook or set of statistical tables; the "degrees of freedom" to be used are given by

$$\nu = n-1.$$

When properly carried out, the chi-square dispersion test is more powerful than the Kolmogorov–Smirnov (K–S) test in most situations, particularly when one wishes to detect either the presence of sources of error additional to the Poisson-type counting error or the occurrence of outliers. Since the K–S test is more difficult to apply, this makes the dispersion test the obvious choice for the given situation.

There is a test commonly known as the chi-square goodness-of-fit test. It is sometimes confused with the chi-square dispersion test because of the similarity in name, but the calculations for it and its properties are totally unconnected with those of the dispersion test or any other test described in the article. It was correctly omitted from consideration since it is not applicable when few data points are available.

The use of the sample range to estimate a standard deviation is a well-known technique. However, the statistic which is calculated from it does not have the χ^2 distribution that would be associated with the sample variance, S^2, calculated in the normal way. While the value calculated using Eq. 8 of the article may be used as a rough guide, no great reliance can be put on it, especially for small sample sizes. If the experimental work is worth doing, then it is also worth fully using the information thus obtained, especially now that simple electronic calculators are so cheap and universally available.

TABLE 1. Number of sample values n required to be reasonably certain (90% certain) of detecting as significant a standard deviation which is k time as large as it should be.

n	k
4	3.7
6	2.6
8	2.2
10	2.0
15	1.7
20	1.6
40	1.4

There is in practice a method of calculating the chi-square dispersion statistic which is algebraically identical to Eq. 7 in the article, but is much simpler for use with calculating machines of even the least sophisticated nature. Calculate

$$T = \Sigma X_i \text{ and } Q = \Sigma X_i^2 .$$

Then

$$\frac{\Sigma(X_i - \bar{X})^2}{\bar{X}} = \frac{(n-1)S^2}{\bar{X}} = \frac{Q - T^2/n}{T/n} .$$

(*Note:* This method of calculating S^2 should not be used in circumstances where n is large and S is much smaller than $\bar{X}$, since rounding errors in the calculations become important.)

No mention is made in the article of the optimal sample size to be used for these tests. In order to decide this it is necessary to know the reason for investigating the problem and the importance of the size of the discrepancy between the actual and the expected variation.

Table 1 may be of some help in determining the minimum sample size required for the dispersion test.

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